# 6.5 Intra- and Intervalley Deformation Potentials for Electrons in GaAs

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## A DEFORMATION POTENTIALS

A deformation potential (DP) is a quantity proportional to a matrix element of an operator belonging to a crystal deformation (which may be caused by pressure, strain, or a phonon displacement) between a final and initial electron or hole state [1]. It usually has the units of eV or eV/Å. The DP is called an intravalley DP if the wave vectors of the final and initial states are within the same electron valley; otherwise, it is called an intervalley DP. Intravalley DPs cause shifts, splittings, and/or intravalley scattering of carriers, whereas intervalley DPs are responsible for the scattering of electrons to a different valley. We note that an intravalley DP can cause an interband splitting, if several valleys (i.e., L or X) are degenerate. A DP is called an intraband DP if the band indices of the final and initial states are the same; otherwise, it is called an interband DP. The DP for an optical gap is the difference of the absolute DPs for the valence and conduction band states and describes the change of the gap due to pressure, strain, or an optical phonon. The bulk of this review deals with intraband intravalley and intervalley DPs for the conduction band of GaAs and for the shifts and splittings of optical gaps. We also mention briefly two-phonon DPs. The intravalley DPs for holes in the  $\Gamma$  and L- valence bands are discussed in the following Datareview by Adachi [2].

Deformation potentials relate shifts and splittings to the elements of the 3×3 strain tensor  $\varepsilon$  defined by  $\vec{r}' = (1 + e)\vec{r}$ , where  $\vec{r}'$  is the strained and  $\vec{r}$  the unstrained coordinate [3,4]. It is convenient (but not necessary) to break up an arbitrary strain into its irreducible components, which are (for a cubic crystal) hydrostatic pressure (with  $\Gamma_1$  symmetry), [100] strain ( $\Gamma_{12}$ ), and [111] strain ( $\Gamma_{15}$ ), see [5]. An experiment, on the other hand, usually applies a stress defined by a 3×3 stress tensor **X** [4,5]. The strain and stress tensors are related through the 6×6 compliance tensor **S** [4], which has three independent components  $S_{11}$ ,  $S_{12}$ , and  $S_{14}$ . Yu and Cardona [5] list the strain tensors for [100] and [111] uniaxial stress. The important conversion factors are:

Hydrostatic pressure:

$$\mathbf{X} = \begin{pmatrix} -P & 0 & 0 \\ 0 & -P & 0 \\ 0 & 0 & -P \end{pmatrix}, \quad \text{Tr } \mathbf{e} = 3\mathbf{e}_H = -3P(S_{11} + 2S_{12}).$$

Tensile uniaxial stress along [100]:

$$\mathbf{X} = \begin{pmatrix} X & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \text{Tr } \mathbf{e} = 3\mathbf{e}_{H} = (S_{11} + 2S_{12})X, \qquad \mathbf{e}_{xx} - \mathbf{e}_{yy} = 3\mathbf{e}_{S} = (S_{11} - S_{12})X.$$

Tensile uniaxial stress along [111]:

$$\mathbf{X} = \frac{X}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \qquad \text{Tr } \mathbf{e} = 3\mathbf{e}_{H} = (S_{11} + 2S_{12})X, \qquad \mathbf{e}_{xy} = \frac{1}{6}S_{44}X.$$

Different conversion factors apply to the biaxial strain found in pseudomorphically strained layers (e.g., AlAs on GaAs or Ge on Si), see [74]. We note that there is also a different convention, where the strain becomes a 6-dimensional array [5]. In this case,  $\varepsilon_4 = S_{44}X/3$ .

# B INTRAVALLEY DEFORMATION POTENTIALS FOR ELECTRONS IN GaAs

#### B1 Γ-Point

The absolute shift of the conduction band minimum under pressure (or strain) is equal to the **absolute** deformation potential a times the trace of the strain tensor e:

$$\Delta E_{\rm CR}(\Gamma) = a \, \text{Tr} \, \boldsymbol{e}. \tag{1}$$

The same a gives in principle the coupling of the electrons with the longitudinal acoustic phonons [6]. However, while the coupling to acoustic phonons is uniquely defined, that to a strain is not for a crystal extending to infinity: an absolute average potential cannot be defined nor its derivative versus strain. In a finite crystal, such a potential can be defined, but depends on the surface orientation (electron affinity, photothreshold) and hence cannot be used to calculate electron-LO phonon coupling constants. Methods to calculate absolute hydrostatic DPs which can be used to determine electron-phonon coupling constants (i.e., including screening) have been given in [7,8,9,10,11,12,13]. Such calculations yield very small values of a for holes at the top of the valence band (less than 1 or 2 eV); hence, it is a good approximation to set a for electrons equal to the DP of the corresponding gap [14,15] which can be obtained easily by measuring luminescence, reflectivity, or absorption under pressure [16,17,18,19,20] (or in pseudomorphically strained quantum wells [21]) or from band structure calcuations of solids under pressure [20,22,23,24,25]. We therefore list in TABLE 1 both the deformation potentials for the gap and the absolute deformation potentials for the conduction band at  $\Gamma$ , which should be used in the calculation of electron-LA phonon interactions. Some methods, e.g., transport measurements, can only determine the magnitude of the DP, but not its sign.

The absolute deformation potentials have been calculated using (now obsolete) empirical pseudopotentials [26] (EPM), *ab initio* pseudopotentials (PSP) based on the local density-functional theory (LDA) [8,9,10,13], and with the fully relativistic linear-muffin-tin orbitals (LMTO) method [7,27] within the atomic sphere approximation (ASA). The most probable value lies between 7 and 9 eV, see TABLE 1. The experimental values were determined from transport measurements [28,29,30,31,32,33,34,35,36,37,38] on high-mobility bulk GaAs samples and 2D-GaAs/AlGaAs heterostructures, free-electron absorption [39,40], deep-level transient spectroscopy (DLTS), luminescence of transition metal impurities in GaAs [41,42], and doping of GaAs with Te [43] or Si [44]. They fall into two groups around 7-9 eV and 13-14 eV, but the values depend somewhat on the model used for fitting the data [45,46,47,48],

for example on the type of wave functions, the screening mechanism, the phonon modes in the AlGaAs barrier [37], or the value of the piezo-electric constant used in the simulation. Since the DP for deep levels may differ from that of the band, it may be difficult to derive absolute DPs from the studies of d-impurities [49]. Not all available experimental data for a are given in TABLE 1.

The coupling of electrons of  $\Gamma_1$  symmetry to optical phonons is forbidden by group theory, at least at or close to  $\Gamma$ . So is their coupling to a shear, i.e., to transverse acoustic phonons or to [111] or [100] strain. Therefore, the optical and transverse acoustic deformation potential constants are zero.

Interband deformation potentials for [111]-strain with  $\Gamma_{15}$  symmetry connect the lowest conduction band at  $\Gamma$ , i.e.,  $\Gamma_1$ , with the p-bonding valence band and the p-antibonding conduction band at  $\Gamma$  with  $\Gamma_{15}$  symmetry [50]. These have been calculated with the EPM and tight binding (LCAO) methods [26] and are important for spin-relaxation phenomena [51] and the recently observed piezooptical activity of GaAs [52].

TABLE 1 Deformation potentials for electrons in GaAs at  $\Gamma$ , in eV.

а	Method	Reference		
Deformation potential for the band gap				
-7.0	absorption	[14,15]		
-7.99	photoluminescence	[21]		
-8.4±0.9	piezo-electroreflectance	[16]		
-8.7±0.3	absorption	[17]		
-8.4±0.2	reflectivity	[20]		
-9.8	absorption	[19]		
-9.9	empirical pseudopotential method (EPM)	[22]		
-8.1	ab initio pseudopotentials	[23]		
-8.3±1	ab initio pseudopotentials	[8,9]		
-9.0±1	linear-muffin-tin orbitals method (LMTO)	[24]		
-8.34	tight binding	[25]		
A	absolute deformation potential for the conduction b	and		
-7.3	ab initio pseudopotentials	[8,9]		
-8.8	LMTO (screened)	[7]		
-8.47	ab initio pseudopotentials	[13]		
±7	transport	[15,28,33]		
±12	transport	[46]		
$\pm (13.5\pm 0.5)$	transport	[45]		
±(9.5±0.5)	transport	[31]		
±8.5	transport	[47]		
±8	transport	[48]		
±17.5	transport	[32]		
±(11±1)	transport (2D)	[30]		
±13.5	transport (2D)	[29]		
-9.3	d impurity levels	[41]		
-7.7	d impurity levels	[42]		
±15.7	infrared absorption	[40]		
±(12±1)	transport (2D)	[34]		

-8.5	doping with Si	[44]
±7	transport (2D)	[35]
±(11.5±0.5)	transport (2D)	[36]

#### B2 L-Point

The four degenerate conduction band states at L split under [111] (but not [100]) uniaxial strain into a triplet and a singlet. The interband splittings, determined by the shear deformation potentials  $\Xi_d$  and  $\Xi_u$  in the Herring-Vogt notation [53], have a 3:–1 ratio.  $\Xi_u$  is also denoted by  $E_2$  [54]. An overview of the different notations has been given by Kane [55]. The associated **interband shift for valley**  $\hat{n}$  for an arbitrary strain (relative to the unstrained energy level) is given by [9,54]

$$\Delta E_{\hat{n}} = E_2 \hat{n} \left( \mathbf{e} - \frac{1}{3} \operatorname{Tr} \mathbf{e} \right) \hat{n}, \tag{2}$$

where e is the strain tensor and the unit vector  $\hat{n}$  is the direction of the valley. The hydrostatic deformation potential a for L-electrons, see Eqn. (1), is also denoted by  $E_I$  and given by

$$a = E_1 = \frac{1}{3}\Xi_u + \Xi_d. \tag{3}$$

It is derived from the average shift of all four states [53]. As in the case of the conduction band at  $\Gamma$ , it is a reasonable approximation to set a equal to the deformation potential for the indirect gap. Then, a can be obtained from band structure calculations under hydrostatic pressure [23,24,56]. An experimental value of a=3 eV was derived from photoreflectance studies of  $GaAs/Al_xGa_{1-x}As$  quantum wells under high pressure [57].  $\Xi_u$  has been calculated using ab initio pseudopotentials [9], LMTO [24], and EPM [58]. Calculations of  $\Xi_d$  and  $\Xi_u$  turn out to be sensitive to the internal displacement parameter  $\zeta$ , which describes the change of the basis vectors with strain [9]. Therefore, the acuracy of the atomic sphere-approximation (ASA) is limited for this case. Experimental data for  $\Xi_u$  are available from piezoresistance [59] and hot-electron photoluminescence measurements [60], seeTABLE 2.

TABLE 2 Deformation potentials for electrons at the L-point in GaAs, in units of eV.

$\Xi_{\mathrm{u}}=E_2$	$a=E_1$	$d_{1o}^{5}$	Method	Reference
14.26			ab initio pseudopotentials	[9]
19.6±3			piezoresistance	[59]
14.5±1.5			hot-electron photoluminescence	[60]
	±9.2	±17	transport	[64]
	-3		photoreflectance	[57]
	-2.1		dielectric theory	[56]
18.5	-3.3		linear-muffin-tin orbitals method (LMTO-ASA)	[24]
	-3.3		ab initio pseudopotentials	[23]
		-20	empirical pseudopotentials (EPM)	[58]
		-32	empirical pseudopotentials (EPM)	[63]
		-21.7	empirical pseudopotentials (EPM)	[61]

The interaction between the electrons at L and optical phonons at  $\Gamma$  is given by the optical deformation potential  $d_{1o}^5$ , which is often written as

$$D = D_{t}K = \frac{d_{1o}^{5}}{a_{0}},\tag{4}$$

where  $a_0$  is the lattice constant [61,62].  $d_{10}^5$  has been calculated using the EPM [58,61,63] and has also been determined from transport measurements [64], seeTable 2.

## B3 X-Point

The deformation potentials  $\Xi_d$  and  $\Xi_u$  (or a,  $E_1$ , and  $E_2$ ) for the three X-valleys are defined just as in the case of the L-valleys, see Eqns. (1) to (3). However, for the X-point, the interband shifts are nonzero for [100] strain and vanish for [111] strain. Experimental values for  $E_2$  have been obtained using hot-electron photoluminescence [60] and 2D-transport under uniaxial stress [65], for a with absorption measurements of the indirect gap at X [17]. AlAs can be studied using time-resolved photoluminescence, since it is indirect [66,67]. The splitting of the X-valleys in GaAs under [100] strain, i.e.,  $E_2$ , has been calculated with pseudopotentials [9], the shifts of the indirect gap at X under hydrostatic pressure relative to the valence band top, i.e., a, with the dielectric theory [56], pseudopotentials [9,23,59], and LMTO [24], as shown in TABLE 3.

TABLE 3 Deformation potentials for electrons at the X-point in GaAs, in units of eV.

$\Xi_{ m u} = E_2$	$a=E_1$	Method	Reference
8.61		ab initio pseudopotentials	[9]
6.5±1		hot-electron photoluminescence	[60]
5.8±0.1		time-resolved photoluminescence (AlAs)	[66]
	±9.3	transport	[64]
6.3		empirical pseudopotentials (EPM)	[59]
9.6±1.8		transport (2D, IV-characteristics)	[65]
	+1.7	absorption	[17]
	+0.6	dielectric theory	[56]
	+1.6	linear-muffin-tin orbitals method (LMTO-ASA)	[24]
	+1.7	ab initio pseudopotentials	[23]
	-0.09	ab initio pseudopotentials	[13]

# **B4** Lines of High Symmetry

Deformation potentials can be defined at any point in the Brillouin zone, not just at points of high symmetry. The corresponding shifts can be measured, for example, using angle-resolved photoemission [68]. The dispersion of DPs in the conduction and valence bands of GaAs, calculated using empirical pseudopotentials, is shown in [58,63,71]. EPM results compare favorably with LMTO results, see [69].

# C DEFORMATION POTENTIALS FOR INTERBAND TRANSITIONS IN GaAs

# C1 $\Gamma$ -Point ( $E_{\theta}$ transition) and indirect transitions

The deformation potentials for the  $E_0$  gap are similar to the absolute deformation potentials for  $\Gamma$  electrons. Therefore, they have been discussed in SECTION B1. Also, the DPs for indirect transitions between  $\Gamma$  and X or L are about the same as the absolute DPs for the electrons at X and L listed in the previous section.

# C2 L-Point ( $E_1$ and $E_1+D_1$ transitions)

Since optical interband transitions near L and along most of the  $\Lambda$  direction produce strong peaks in the dielectric function (the so-called  $E_I$  and  $E_I + \mathbf{D}_I$  transitions), it is also of interest to know the effect of hydrostatic pressure, uniaxial strain (acoustic phonons), and of optical phonons [70] at  $\Gamma$  on these transitions. The shifts of  $E_I$  and  $E_I + \Delta_I$  due to hydrostatic pressure

are described by the **hydrostatic deformation potential**  $a = \frac{D_1^1}{\sqrt{3}}$ , see TABLE 4.

$a = D_1^1 / \sqrt{3}$	Method	Reference
-4.3±0.4	piezo-reflectance	[16]
-6.9±0.2	reflectivity	[20]
-8.3	empirical pseudopotential method (EPM)	[16]
-5.5	empirical pseudopotential method (EPM)	[22]
-5.82	linear-muffin-tin orbitals method (LMTO-ASA)	[20]
-4.8±0.5	ellipsometry under uniaxial stress	[71]

TABLE 4 Hydrostatic deformation potentials for  $E_1$  and  $E_1+\Delta_1$  transitions in GaAs, in eV.

We note that there are three contributions to the splittings of the  $E_1$  and  $E_1 + \mathbf{D}_1$  doublet due to uniaxial strain [71]:

- 1. The four degenerate <111> directions split under [111] (but not [100]) strain (interband splitting).
- 2. The doubly degenerate valence band at L (neglecting spin orbit splittings) splits under [111] and [100] strain (intraband splitting). If spin-orbit splittings are included, the (apparent) value of  $\mathbf{D}_{l}$  will change.
- 3. There also are spin-exchange terms, which we neglect here, since they are small. Diagrams showing the shifts and splittings of the different bands are given in [72,73]. In the previous (2nd) edition of this book, we discussed interband and intraband splittings in our Datareview. In this (3rd) edition, the intraband splittings are discussed in the Datareview by Adachi [2], since they are related to the hole deformation potentials  $D_3^5$  and  $D_3^3$ . The optical deformation potentials  $d_{10}^5$  and  $d_{30}^5$  are also given by Adachi [2].

A [111] strain shifts the [111] -direction from the three  $\langle 1\,\overline{1}\,\overline{1}\rangle$ -directions in a manner similar to that in Eqn. (2), with the energy shifts in a 3:-1 ratio. The corresponding deformation

potentials are usually given in Kane's notation [55] as  $D_1^5$ , whereby  $D_1^5$  must be plugged into Eqn. (2) through the substitution [55]

$$E_2 = \frac{\sqrt{3}}{2} D_1^5. {5}$$

to find the strain splittings of the  $E_I$  interband transitions.  $D_1^5$  has been determined experimentally using electroreflectance [16,73] and ellipsometry [71] under uniaxial stress and calculated using the EPM [16,22,58], see TABLE 5.

The doubly-degenerate valence bands at L (or along  $\Lambda$ ) are split by the [111] strain. A [100] strain does not remove the degeneracy of the four L-points, but it also splits the doubly-degenerate valence band maximum at L. These intraband splittings are described by the deformation potentials  $D_3^5$  and  $D_3^3$ , see [2]. The resulting eigenvalues of the effective strain Hamiltonian (i.e., the energies of the  $E_I$  and  $E_I$ + $\mathbf{D}_I$  transitions under strain) are given in [71,72,74]. They are presumed to be correct in this work as well as in [72,73,74], but there are misprints in [16,71].

For [100] stress, these eigenvalues are [16,73,74]

$$E_1, E_1 + \Delta_1 = E_1^0 + \frac{\Delta_1}{2} + \Delta E_H \pm \sqrt{\left(\frac{\Delta_1}{2}\right)^2 + \left(\Delta E_S^{\text{intra}}\right)^2}, \quad \text{where} \quad \Delta E_H = \sqrt{3}D_1^1 \boldsymbol{e}_H, \quad \Delta E_S^{\text{intra}} = \sqrt{6}D_3^3 \boldsymbol{e}_S.$$

In the large-shear approximation, the intraband shear splittings are much larger than the spinorbit splitting ( $\Delta E_S^{\text{intra}} >> \Delta_1$ ). This approximation is appropriate for Si [72]. For GaAs, it is usually feasible to use the small-shear approximation ( $\Delta E_S^{\text{intra}} << \Delta_1$ ), see [16,71], instead of the accurate expression above.

For [111] stress, the  $E_I$  and  $E_I + \mathbf{D}_I$  transitions split into a singlett and a triplett. The energies of the singlett are [16,72,73]

$$E_1^S = E_1^0 + \Delta E_H + \Delta E_S^{\text{inter}}, \qquad \left(E_1 + \Delta_1\right)^S = E_1^0 + \Delta_1 + \Delta E_H + \Delta E_S^{\text{inter}},$$

where the hydrostatic shift  $\Delta E_H$  is defined just like for [100] stress and the interband shift of the singlet is  $\Delta E_S^{\text{inter}} = \sqrt{3}D_1^5 \boldsymbol{e}_{xy}$ . The triplett energies in the small-shear approximation are [16,71]

$$E_1^S = E_1^0 + \Delta E_H - \frac{1}{3} \Delta E_S^{\text{inter}} - \frac{\left(\Delta E_S^{\text{intra}}\right)^2}{\Delta_1}, \quad \left(E_1 + \Delta_1\right)^S = E_1^0 + \Delta_1 + \Delta E_H - \frac{1}{3} \Delta E_S^{\text{inter}} + \frac{\left(\Delta E_S^{\text{intra}}\right)^2}{\Delta_1},$$

where the intraband shear splitting is  $\Delta E_s^{\text{intra}} = \sqrt{\frac{8}{3}} D_3^5 \boldsymbol{e}_{xy}$ . The analogous results for the large-

shear case are [71]:  $E_1^S = E_1^0 + \Delta E_H - \frac{1}{3} \Delta E_S^{\text{inter}} \pm \Delta E_S^{\text{intra}}$ .

## C3 X-Point

The  $X_5 \to X_1$ -gap at the X-point is also split by a [100] strain (but not by a [111] strain) in a manner somewhat similar to the splitting of the  $E_I$ -gap by [111] strain. The interband splitting of the [100] valley from [010] and [001] is given by Eqn. (2), where  $E_2$  in Kane's notation [55] is replaced by

$$E_2 = \sqrt{\frac{3}{2}}D_1^3. {(6)}$$

Values of  $D_1^3 = -10.4$  eV have been obtained with the EPM method [58]. The intraband splittings in the valence band at X are discussed in [2].

TABLE 5 Acoustic Deformation potentials  $D_1^5$  describing the interband splittings of the  $E_1$  and  $E_1+\Delta_1$  transitions due to a [111] shear, see Eqn. (5).

$D_1^5$	Method	Reference
9.2±1.0	piezo-electroreflectance (77 K)	[16]
8.5±0.8	piezo-electroreflectance (300 K)	[73]
12.0±0.7	ellipsometry under uniaxial stress (300 K)	[71]
7.9±0.5	EPM(L)	[16]
8.3	$EPM\;(L)$	[22]
5.5	$EPM(\Lambda)$	[22]
12.4	$EPM\;(L)$	[58,71]
9	$EPM(\Lambda)$	[58]

## D INTERVALLEY DEFORMATION POTENTIALS FOR ELECTRONS IN GaAs

#### **D1** Introduction and Notation

A scattering process of an electron by a phonon is called an intervalley scattering (IVS) process, when it connects two different minima in the conduction band or maxima in the valence band (e.g.,  $\Gamma \to L$ ,  $\Gamma \to X$ ,  $L \to X$ ,  $X \to X'$ ,  $L \to L'$ , etc). In GaAs, for these processes the wavelength of the scattered phonon is of the order of the lattice constant, i.e., the crystal momentum transfer Q is larger than about one sixth of a reciprocal lattice vector. According to Eqn. (3.6.7) of Conwell's book [62], the matrix element for this process is proportional to the intervalley deformation potential D (IDP, in units of eV/Å). The expression also contains the intervalley phonon energy  $\Omega$  and the mass M=Vr of the primitive cell (i.e., the mass of one Ga plus one As atom) [75].

The selection rules for IVS processes have been given by Birman, Lax, and Loudon [76]. They are discussed in [77] in the light of new arguments about the symmetries of electrons and phonons at the X- and L-points. Selection rules along lines of high symmetry are given in [75]. The conditions for these selection rules to be valid are usually not fulfilled, since energy conservation rules out scattering processes between electrons exactly at high-symmetry points. Nevertheless, it is generally assumed that the matrix elements for intervalley transitions are nearly independent of the phonon wave vector Q. Therefore, the matrix

elements can be integrated over all possible final states in a spherical energy band resulting in Conwell's expression for the IVS time  $\tau$ :

$$\frac{1}{\tau} = \frac{N_v D^2 m_v^{3/2}}{\sqrt{2}\pi\hbar^2 \rho \Omega} \left[ (N+1)\sqrt{E - \Delta E - \Omega} + N\sqrt{E - \Delta E + \Omega} \right],\tag{8}$$

where  $N_V$  is the number of (final) valleys,  $m_V$  the effective mass of the final valley, r the density of the crystal, N the occupation number of the intervalley phonon, E the energy of the electron in the initial valley, and  $\Delta E$  the energy separation between the two valleys. See Eqn. (3.6.14) in [62] and also [77,78].

## **D2** Theoretical Results

At least five independent calculations of intervalley deformations potentials for GaAs using empirical band structure methods have appeared in the literature [75,77,79,80,81,82,83]. They are based on parametrized lattice dynamical models for the phonon eigenvectors (note that the LO and LA phonons at X the eigenvectors are fixed by symmetry) and empirical pseudopotential or tight-binding electron wave functions. More recently, two *ab initio* calculations have been performed [84,85,86,87]. Their results are given in TABLE 6. The agreement between the different methods is rather good. The IDPs given here are for the screened case [79,80]. Values calculated from the EPM using local and nonlocal pseudopotentials are in good agreement [82,83]. There is, however, some disagreement how the form factors should be extrapolated to q=0 [75,83,88]. IDPs for hole-phonon scattering have also been reported [84,85,86].

 $X \to \overline{X'}$  $\Gamma \to L$  $L \rightarrow L'$  $L \rightarrow X$  $\Gamma \to X$ Ref. LA+LO LO LA+LO LO LA+LOTO TA TΑ 2.6±0.3 4.9 0.8 0.8 1.8  $2.8\pm0.1$ 1.2 1.8 [77] 3.4 5.0±0.5 [80] 3.2 [81] 3.7 4.0 1.0 6.3 0.0 3.7 2.2 [82] 3.4 3.4 3.8 2.9 2.8 [83] 3.86 4.97 0.36 2.32 [84] 4.134 [85]

TABLE 6 Calculated intervalley deformation potentials for GaAs in units of eV/ Å.

Herbert [79] has found that the IDP dependends on the phonon wave vector Q. Detailed plots of the dispersion of the IDPs are given in [75,89]. There it has been shown that the transverse acoustic phonon needs to be included for  $\Gamma \to X$  scattering, wherease the assumption of k-independent matrix elements gives smaller (but still significant) errors for  $\Gamma \to L$  scattering. Because of this Q-dependence, IDPs extracted from experiments are effective values depending on the experimental conditions, like the bath temperature [90], the direction of the electric field, or the incident laser energy [78,91,92].

## D3 Experimental Results

Estimates of IDPs have been obtained from a number of different optical and electrical measurements. The evaluation of the data is usually difficult, as several different scattering

mechanisms (carrier-carrier, intervalley, intravalley, impurity, and possibly alloy scattering [93]) are possible and compete with each other. Usually, a numerical model based on rate equations or Monte-Carlo simulations is fitted to the data. There are almost as many sets of IDPs as there are experiments, disagreeing with each other by more than one order of magnitude. We have therefore selected only a few papers which we believe to give the most reliable results.

The IDP for  $\Gamma \to L$  scattering has been determined by various recent optical experiments, see: Subpicosecond luminescence spectroscopy [94,95], hot-electron photoluminescence [96,97,98,99,100,101,102,103,104,105,106], time-resolved Raman spectroscopy [107], broadening of the direct exciton under hydrostatic pressure [108,109], and infrared four-wave mixing [110]. Mirlin and coworkers [60] have used the method of hot-electron luminescence under uniaxial stress to determine the  $L \to L'$  IDP relative to the  $\Gamma \to L$ . A better picture of scattering has evolved recently, since Monte Carlo [111,112,113,114,115,116,117,118,119] are better able to keep track of the many particles generated in femtosecond laser pump-and-probe and four-wave mixing experiments [90,95,120,121,122,123,124,125]. An important issue is also the inclusion of the full band structure for electrons and phonons [126] and the Q-dependence of the electron-phonon matrix elements [78,83]. The values obtained by Collins and Yu [127] for the  $\Gamma \to L$  process using nonequlibrium phonon spectroscopy have been criticized in [128].

 $\Gamma \to L$  IDPs obtained from Monte-Carlo [33,81,129,130,131,132,133] and Green's function [134,135] simulations of electrical measurements [136,137,138] are in the same range of values, but the velocity-field curves are not very sensitive to this parameter [117]. Similar values are listed in [139] from an analysis of impact ionization coefficients. Combined optical and electrical methods (transient grating and noise experiments) have been used in [140], resulting in a lower  $\Gamma \to L$  IDP. The IDP for  $\Gamma \to X$  scattering is generally assumed to be larger than the  $\Gamma \to L$  IDP. Little experimental evidence exists about IDPs for scattering between other valleys, see TABLE 7.

When experimental IDPs are given in the literature, it is generally assumed that the band structure of a valley can be described in an effective mass approximation. This is certainly not the case for the *X*-valley with its camelback structure [141,142], where f and g processes (as in silicon) are possible. Also, because of the camelback structure it is not clear if there 3 or 6 *X*-valleys.

## E ELECTRON-TWO PHONON INTERACTIONS

Processes in which an electron scatters through creation or annihilation of two phonons in a single (renormalized) vertex can be important in transport phenomena [143,144] and in second-order Raman scattering (i.e., by two phonons). The interaction constant is usually represented by a deformation potential  $D_i$ , which describes the second derivative of the interaction energy with respect to the phonon displacement, and multiplied by the square of the lattice constant in order to obtain the dimensions of an energy (eV). From resonant Raman scattering deformation potentials corresponding to the  $E_0$  and  $E_1$  gaps have been obtained for

GaAs, see [70], Table 2.11. They represent the effect of combinations of two phonons of  $\Gamma_1$  symmetry for the DP  $D_I$  (for the  $E_0$  and  $E_I$  gaps), or of  $\Gamma_{15}$  symmetry for  $D_{15}$  ( $E_0$  gap) or for  $D_{3o}^5$  ( $E_I$  gap).

## F CONCLUSIONS

An empirical rule due to W. Paul [145] states that DPs for the same type or process are similar for most semiconductors. Therefore, the DPs listed in this review can be used (with reasonable accuracy) for materials similar to GaAs, such as Ge, InP, AlAs, and, possibly, even Si. By the same token, if we do not list a particular DP for GaAs, values for similar semiconductor materials can be used.

A few DPs, such as *a* describing the shifts of the direct gap with hydrostatic pressure, are known fairly accurately. Most DPs, however, particularly absolute DPs and intervalley DPs, are difficult to calculate or measure. Therefore, there is considerable scatter in the literature. Uncertainties of 50% or more are common.

TABLE 7 Experimentally determined absolute values of IDPs in GaAs, in units of eV/Å.

$\Gamma \to L$	$\Gamma \to X$	$L \rightarrow L'$	$X \to L$	$X \to X'$	Reference
6.5±1.5					[94]
3.5					[100]
7±2					[107]
8±1	15±3	5±1			[60,96,97]
9±2					[106]
>5					[116]
9					[117]
7±1					[110]
<1.5	11±1		2.75±0.2		[127]
3.25					[81]
10	10	10	5	7	[33,131,135]
10	10	10	9	9	[129,130]
2.85	10	5	3.16	10	[134]
1.8	10	5	1	10	[132]
5	8		1.810	10	[139]
3					[140]
	10±1				[108]
	4.8±0.3	·			[109]
5±0.5					[118]

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- [1] For a general introduction, see G.L. Bir, G.E. Pikus [Symmetry and strain-induced effects in semiconductors, (Wiley, New York, 1974)]
- [2] S. Adachi [ Datareview in this book: 6.6 Intravalley deformation potentials for holes in GaAs
- [3] L.D. Landau, E.M. Lifshitz [Theory of Elasticity (Pergamon, New York, 1986)]
- [4] J.F. Nye [ Physical Properties of Crystal's (Clarendon, Oxford, 1985) ]
- [5] P.Y. Yu, M. Cardona [Fundamentals of Semiconductors (Springer, Berlin, 1996)]
- [6] J. Bardeen, W. Shockley [ Phys. Rev. (USA) vol.80 (1950) p.72-80 ]
- [7] M. Cardona, N.E. Christensen [ Phys. Rev. B (USA) vol.35 (1987), p.6182-94; vol.36 (1987) p.2906 (E) ]
- [8] C.G. van de Walle, R.M. Martin [ Phys. Rev. Lett. (USA) vol.62 (1989), p.2028-31 ]
- [9] C.G. van de Walle [ Phys. Rev. B (USA) vol.39 (1989) p.1871-83 ]
- [10] R.D. King-Smith, R.J. Needs [J. Phys.: Condensed Matt er vol.2 (1990) p.3431-44]
- [11] R. Resta, L. Colombo, S. Baroni [ Phys. Rev. B (USA) vol.41 (1990) p.12358-61 ]
- [12] A. Franceschetti, S.-H. Wei, A. Zunger [Phys. Rev. B (USA) vol. 50 (1994) p.17797-801]
- [13] R.A. Casali [ Solid State Commun. (USA) 1996 (in press) ]
- [14] H Ehrenreich [ Phys. Rev. (USA) vol.120 (1960), p.1951-63 ]
- [15] D.L. Rode [ Phys. Rev. B (USA) vol.2 (1970) p.1012-24 ]
- [16] M. Chandrasekhar, F.H. Pollak [ Phys. Rev. B (USA) vol.15 (1977) p.2127-44 ]; also P.J. Melz (unpublished)
- [17] A.R. Goñi, K. Strössner, K. Syassen, M. Cardona [ Phys. Rev. B (USA) vol.36 (1987) p.1581-7 ]
- [18] D. Olego, M. Cardona, H. Müller [ Phys. Rev. B (USA) vol.22 (1980) p.894-903 ]
- [19] B. Welber, M. Cardona, C.K. Kim, S. Rodriguez [ Phys. Rev. B (USA) vol.12 (1975) p.5729-38 ]
- [20] M. Hanfland, K. Syassen, N.E. Christensen [J. Phys. (France) vol.45 (1984) Suppl. C8 p.57-60 ]
- [21] V.A. Wilkinson, A.D. Prins, J.D. Lambkin, E.P. O'Reilly, D.J. Dunstan, L.K Howard, M.T. Emeny [Phys. Rev. B (USA) vol.42 (1990) p.3113-9 ]
- [22] Y.F. Tsay, B. Bendow [ Phys. Rev. B (USA) vol.16 (1977) p.2663-75 ]
- [23] K.J. Chang, S. Froyen, M.L. Cohen, [Solid State Commun. (USA) vol.50 (1984) p.105-7
- [24] N.E. Christensen [ Phys. Rev. B (USA) vol.30 (1984) p.5753-65 ]
- [25] C. Priester, G. Allan, M. Lannoo [Phys. Rev. B (USA) vol.37 (1988) 8519-22]
- [26] A. Blacha, H. Presting, M. Cardona [phys. stat. solidi (b) (Germany) vol.126 (1984) p.11-36
- [27] A. Vergès, D. Glötzel, M. Cardona, O.K. Anderson [phys. stat. solidi (b) (Ger many) vol.113 (1982) p.519-534 ]
- [28] W. Walukiewicz, H.E. Ruda, J. Lagowski, H.C. Gatos [ Phys. Rev. B (USA) vol.32 (1985) p.2645-6]
- [29] E.E. Mendez, P.J. Price, M. Heiblum [Appl. Phys. Lett. (USA) vol.45 (1984) p.294-6]
- [30] K. Hirakawa, H. Sakaki [ Appl. Phys. Lett (USA) vol.49 (1986) p.889-91 ]
- [31] D.C. Look, P.C. Colter [ Phys. Rev. B (USA) vol.28 (1983) p.1151-3 ]
- [32] H.J. Lee, J. Basinski, L.Y. Juravel, J.C. Woolley [Can. J. Phys. (Canada) vol. 57 (1978) p.233-42]; H.J. Lee, J.C. Woolley [Can. J. Phys. (Canada) vol.57 (1978) 1929-33]

- [33] M.A. Littlejohn, J.R. Hauser, T.H. Glisson [J. Appl. Phys. (USA) vol.48 (1977) p.4587-90 ]
- [34] I. Gorczyca, J. Krupski [ Phys. Rev. B (USA) vol.52 (1995) p.11248-58 ]; I. Gorczyca, J. Krupski [ Jpn. J. Appl. Phys. (Japan) vol.32 (????) Suppl.1 p.135-7 ] QC1.J273x
- [35] A.J. Vickers [ Phys. Rev. B (USA) vol.46 (1992) p.13313-8 ]
- [36] I. Gorcyca, T. Suksi, E. Litwin-Staszewska, L. Dmowski, J. Krupski [ Phys. Rev. B (USA) vol.46 (1992) p.4328-31 ]
- [37] T. Kawamura S. Das Sarma R. Jalabert J.K. Jain [ Phys. Rev. B (USA) vol.42 (1990) p.5407-10 ]
- [38] V.I. Kadushkin, A.A. Denisov, S.V. Kolosova [ Sov. Physics-Semicond. (USA) vol.23 (1989) p.1067-9 ]
- [39] W.G. Spitzer, Y.M. Whelan [ Phys. Rev. (USA) vol.114 (1959) p.59-63 ]
- [40] P. Pfeffer, I. Gorczyca, W. Zawadzki [Solid State Commun. (USA) vol. 51 (1984) p.178-83]
- [41] D.D. Nolte, W. Walukiewicz, E.E. Haller [ Phys. Rev. Lett. (USA) vol.59 (1987) p.501-4; Phys. Rev. B (USA) vol.36 (1987) p.9374-7 ]
- [42] L. Samuelson, S. Nilsson [ J. Lumin. (Netherlands) vol. 40&41 (1988) p.127-8 ]
- [43] J. Bak-Misiuk, M. Leszczynski, J. Dormagala, Z. Zytkiewicz [ J. Appl. Phys. (USA) vol.78 (1995) p.6994-8 ]
- [44] M. Leszczynski, J. Bak-Misiuk, J. Domagala, J. Muszalski, M. Kaniewska, J. Marczewski [ Appl. Phys. Lett. (USA) vol.67 (1995) p.539-41 ]
- [45] P.J. Price [ Phys. Rev. B (USA) vol. 32 (1985) p.2643-4 ]
- [46] B. Vinter [ Phys. Rev. B (USA) vol. 33 (1986) p.5904-5 ]
- [47] X.L. Lei [ J. Phys. C (UK) vol. 18 (1985) p.L593-8 ]
- [48] Y. Okuyama, N. Tokuda [ Phys. Rev. B (USA) vol. 40 (1989) p.9744-50 ]
- [49] V. Kostin, E.B. Osipov, N.A. Osipova [Semiconductors (USA) vol.27 (1993) p.962-3]
- [50] M. Cardona, N.E. Christensen, G. Fasol [ Phys. Rev. B (USA) vol.38 (1988) p.1806-27 ]
- [51] R. Ranvaud, H.-R. Trebin, F.H. Pollak, U. Rössler [ Phys. Rev. B (USA) vol.20 (1979) p.701-15 ]
- [52] B. Koopmans, P. Etchegoin, P. Santos, M. Cardona [ Solid State Commun. (USA) vol.97 (1996) p.261-6 ]
- [53] C. Herring, E. Vogt [ Phys. Rev. (USA) vol.101 (1956) p.944-61 ]
- [54] H. Brooks [in 'Advances in Electronics and Electron Physics' Ed. L. Marton (Academic, New York, 1955) vol.7 p.85-182 ]
- [55] E.O. Kane [ Phys. Rev. (USA) vol.178 (1969) p.1368-98 ]
- [56] D.L. Camphausen, G.A.N Connell, W. Paul [ Phys. Rev. Lett. (USA) vol.26 (1971) p.184-88
- [57] A. Kangarlu *et al.* [ Phys. Rev. B (USA) vol.38 (1988) p. 9790-6 ]
- [58] H. Presting [ Hyper-Raman-scattering and electronic properties in binary crystals, Ph.D. thesis, Universität Stuttgart, Germany, 1985 (unpu blished) ]
- [59] D.E. Aspnes, M. Cardona [ Phys. Rev. (USA) vol.17 p.741-51 ]
- [60] D.N. Mirlin, V.F. Sapega, I.Ya. Karlik, R. Katilius [ Solid State Commun. (USA) vol.61 (1987) p.799-802 ]
- [61] W. Pötz, P. Vogl [ Phys. Rev. B (USA) vol.24 (1981) p.2025-37 ]
- [62] E.M. Conwell [ High Field Transport in Semiconductors (Academic, New York, 1967) p.151

- [63] R. Trommer [Raman-scattering in gallium arsenide, Ph.D. thesis, Universität Stuttgart, Germany, 1977 (unpublished)]
- [64] L. Reggiani [ in 'Hot-Electron Tran sport in Semiconductors' Ed. L. Reggiani (Springer, Berlin, 1985) p.80 ]
- [65] S.S. Lu, K.R. Lee, K.H. Lee, M.I. Nathan, M. Heiblum, S.L. Wright [J. Appl. Phys. (USA) vol.67 (1990) p.6360-7]
- [66] S. Charbonneau, J.F. Young, P.T. Coleridge, B. Kettles [ Phys. Rev. B (USA) vol.44 (1991) p.8312-4 ]
- [67] J.Y. Young, S. Charbonneau, P.T. Coleridge [Phys. Rev. B (USA) vol.42 (1990) p.11434-7]
- [68] A. Stampfl *et al.* [ Physica Scripta (Sweden) vol.41 (1990) p.617-20 ]; A. Stampfl *et al.* [ J. Vac. Sci. Technol. A (USA) vol.7 (1989) p.2525-31 ]
- [69] S. Zollner, U. Schmid, N.E. Christensen, C.H. Grein, M. Cardona, L. Ley [ Proc. 20th Int. Conf. on the Physics of Semiconductors, Thessaloniki, Greece, 6-10 Aug 1990, Eds.
   E.M.Anastassakis, J.D. Joannopoulos (World Scientific, Singapore, 1990) vol.2 p.1735-8 ]
- [70] M. Cardona [ in 'Light Scattering in Solids II' Eds. M. Cardona G. Güntherodt (Springer, Berlin, 1982) p.19 ]
- [71] P. Etchegoin, J. Kircher, M. Cardona, C. Grein, E. Bustarret [ Phys. Rev. B (USA) vol.46 (1992) p.15139-49 ]; P. Etchegoin, J. Kircher, M. Cardona, C. Grein [ Phys. Rev. B (USA) vol.45 (1992) p.11721-35 ]
- [72] P. Etchegoin, J. Kircher, M. Cardona [ Phys. Rev. B (USA) vol.47 (1993) p.10292-10303 ]
- [73] F.H. Pollak, M. Cardona [ Phys. Rev. (USA) vol.172 (1968) p.816-37 ]; F.H. Pollak [ Surf. Sci. (Netherlands) vol.37 (1973) p.863-895 ]
- [74] R. Lange, K.E. Junge, S. Zollner, S.S. Iyer, A.P. Powell, K. Eberl [ J. Appl. Phys. (USA) vol.80 (Oct. 15 1996) in print ]
- [75] S. Zollner, Sudha Gopalan, M. Card ona [ J. Appl. Phys. (USA) vol.68 (1990) p.1682-93 ]
- [76] J.L. Birman, M. Lax, R. Loudon [ Phys. Rev. (USA) vol.145 (1966) p.620-2 ]
- [77] S. Zollner, Sudha Gopalan, M. Cardona [Appl. Phys. Lett. (USA) vol.54 (1989) p.614-16. NB: The error in Table I of this reference is corrected in TABLE 6.]
- [78] S. Zollner, Sudha Gopalan, M. Cardona [ Semicond. Sci. Technol. (UK) vol.7 (1992) B137-43 ]
- [79] D.C. Herbert [ J. Phys. C (UK) vol.6 (1973) p.2788-810 ]
- [80] W. Fawcett, D.C. Herbert [ J. Phys. C. (UK) vol.7 (1974) p.1641-54 ]
- [81] S. Krishnamurthy, A. Sher, A.-B. Chen [Appl. Phys. Lett. (USA) vol.53 (1988) p.1853-5]
- [82] S.N. Grinyaev, G.F. Karavaev, V.G. Tyuterev [ Sov. Phys. Semicond. (USA) vol.23 (1989) p.905-7 ]; S.N. Grinyaev, G.F. Karavaev, V.G. Tyuterev, V.A. Chaldyshev [ Sov. Phys. Solid State (USA) vol.30 (1988) p.1586-8 ]
- [83] M.V. Fischetti, J.M. Higman [ in 'Monte Carlo Device Simulation: Full Band and Beyond' Ed. K. Hess (Kluwer, Boston, 1991) p.123-160 ]
- [84] S. Krishnamurthy, M. Cardona [ J. Appl. Phys. (USA) vol.74 (1993) p.2117-9 ]
- [85] J.Q. Wang, Z.Q. Gu, M.F. Li, W.Y. Lai [ Phys. Rev. B (USA) vol.46 (1992) 12358-64 ]
- [86] J.Q. Wang B.Y. Gu [ J. Phys. Condens. Matter (UK) vol.5 (1993) p.647-62 ]
- [87] J.Q. Wang, Z.Q. Gu, M.F. Li, W.Y. Lai [ Commun. Theor. Phys. (China) vol.20 (1993) p.159-70 ]
- [88] S. Bednarek, U. Rössler [Phys. Rev. Lett. (USA) vol.48 (1982) 1296]
- [89] S. Zollner, Sudha Gopalan, M. Cardona [ 'Phonons 89' Proc. 3rd Int. Conf. on Phonon Physics and 6th Int. Conf. on Phonon Scattering in Condensed Matter, Heidelberg, Germany,

- 21-25 Aug 1989, Eds. S. Hunklinger, W. Ludwig, G. Weiss (World Scientific, Singapore, 1990) vol.2 p.787-9 ]
- [90] M.A. Cavichia, W. Wang, R.R. Alfano [in 'Hot Carriers in Semiconductors' Ed. K. Hess (Plenum, New York, 1996), p.365-8]
- [91] S. Zollner, Sudha Gopalan, M. Cardona [ Proc. SPIE (USA) vol.1282 (1990) p.78-85 ]
- [92] S. Zollner, Sudha Gopalan, M. Cardona [ Solid State Commun. vol.76 (1990) p.877-90 ]
- [93] S. Zollner, C.H. Grein, M. Cardona [ Proc. SPIE (USA) vol.1677 (1992) p.75-84 ]; C.H. Grein, S. Zollner, M. Cardona [ Phys. Rev. B (USA) vol.44 (1991) p.12761-8 ]
- [94] J. Shah, B. Deveaud, T.C. Damen, A.C. Gossard, W.T. Tsang, A.C. Gossard, P. Lugli [ Phys. Rev. Lett. (USA) vol.59 (1987) p.2222-5 ]
- [95] H. Kurz [ Semicond. Sci. Technol. (UK) vol.7 (1992) B124-9 ]
- [96] V.D. Dymnikov, D.N. Mirlin, L.P. Nikitin, V.I. Perel', T.T. Reshina, V.F. Sapega [ Sov. Phys.-JETP (USA) vol.53 (1981) p.912-8 ]
- [97] D.N. Mirlin, I.Ya. Karlik, V.F. Sapega [Solid State Commun. (USA) vol.65 (1988) p.171-2]
- [98] D.N. Mirlin, P.S. Kop'ev, I.I. Reshina, V.F. Sapega, A.A. Sirenko [ Proc. 20th Int. Conf. on the Physics of Semiconductors, Thessaloniki, Greece, 6-10 Aug 1990, Eds.
   E.M.Anastassakis, J.D. Joannopoulos (World Scientific, Singapore, 1990) vol.2 p.1037-44 ]
- [99] R.G. Ulbrich, J.A. Kash, J.C. Tsang [ Phys. Rev. Lett. (USA) vol. 62 (1989) p.949-52 ]
- [100] J.A. Kash, R.G. Ulbrich, J.C. Tsang [Solid-State Electron. (UK) vol. 32 (1989) p.1277-82 ] and J.A. Kash [private communication]
- [101] M.A. Alekseev, D.N. Mirlin [ Phys. Rev. Lett. (USA) vol.65 (1990) p.274 ]
- [102] J.A. Kash, J.C. Tsang, R.G. Ulbrich [Phys. Rev. Lett. (USA) vol.65 (1990) p.275]
- [103] J.A. Kash, J.C. Tsang [in 'Light Scattering in Solids VI' Eds. M. Cardona G. Güntherodt (Springer, Berlin, 1991) p.423-518 ]
- [104] W. Hackenberg, G. Fasol [Solid-State Electron. (UK) vol.32 (1989) p.1247-52]
- [105] G. Fasol, W. Hackenberg, H.P. Hughes, K. Ploog, E. Bauser, H. Kano [Phys. Rev. B (USA) vol.41 (1990) p.1461-78]; W. Hackenberg, G. Fasol [Appl. Phys. Lett. (USA) vol.57 (1990) p.174-6]
- [106] W. Hackenberg, G. Fasol, H. Kano [Semicond. Sci. Technol. (UK) vol.7 (1992) B26-8]
- [107] Dai-sik Kim, P.Y. Yu [ Phys. Rev. Lett. (USA) vol. 64 (1990) p.946-9 ]; Dai-sik Kim, P.Y. Yu [ Phys. Rev. B (USA) vol.43 (1991) p.4158-69 ]
- [108] S. Satpathy, M. Chandrasekhar, H.R. Chandrasekhar [ Proc. 20th Int. Conf. on the Physics of Semiconductors, Thessaloniki, Greece, 6-10 Aug 1990, Eds. E.M.Anastassak is, J.D. Joannopoulos (World Scientific, Singapore, 1990) vol.2 p.1521-4 ]; S. Sathpathy, M. Chandrasekhar, H.R. Chandrasekhar, U. Venkateswaran [ Phys. Rev. B (USA) vol.20 (1991) p.11339-44 ]
- [109] A.R. Goñi, A. Cantarero, K. Syassen, M. Cardona [ Phys. Rev. B (USA) vol.41 (1990) p.10111-9]; G.H. Li, A.R. Goñi, K. Syassen, M. Cardona [ Phys. Rev. B (USA) vol.49 (1994) p.8017-23]
- [110] K. Kash, P.A. Wolff, W.A. Bonner [Appl. Phys. Lett. (USA) vol.42 (1983) p.173-5]
- [111] K. Hess [ Monte Carlo Device Simulation: Full Band and Beyond (Kluwer, Boston, 1991) ]
- [112] C. Moglestue [ Monte Carlo Simulation of Semiconductor Devices (Chapman&Hall, London, 1993) ]
- [113] M.J. Kann, A.M. Kriman, D.K. Ferry [Solid-State Electron. (UK) vol.32 (1989) p.1831-6]
- [114] L. Rota, P. Lugli [ Solid-State Electron. (UK) vol. 32 (1989) p.1423-8 ]
- [115] M.A. Osman, H.L. Grubin [Phys. Rev. B (USA) vol.39 (1989) p.10969-72]

- [116] D.W. Bailey, C.J. Stanton, K. Hess, M.J. LaGasse, R.W. Schoenlein, J.G. Fujimoto [ Solid-
- State Electron. (UK) vol.32 (1989) 1491-6]
- [117] D.W. Bailey, C.J. Stanton, M.A. Artaki, K. Hess, F.W. Wise, C.L. Tang [Solid-State Electron. (UK) vol.31 (1988) p.467-70 ]
- [118] U. Hohenester, P. Supancic, P. Kocevar, X.Q. Zhou, U. Lemmer, G.C. Cho, W. Kütt, H. Kurz [Semicond. Sci. Technol. (UK) vol.7 (1992) B176-9]
- [119] D.K. Ferry, E.D. Grann, K.T. Tsen [in 'Hot Carriers in Semiconductors' Ed. K. Hess (Plenum, New York, 1996), p.81-84]
- [120] A. Katz, R.R. Alfano [Appl. Phys. Lett. (USA) vol.53 (1988) p.1065-7]
- [121] P.C. Becker *et al.* [ Appl. Phys. Lett. (USA) vol.53 (1988) p.2089-90 ]
- [122] W.Z. Lin R.W. Schoenlein, J.G. Fujimoto, E.E. Ippen [ IEEE J. Quantum Electron. (USA) vol. 24 (1988) p.267-75 ]
- [123] W.B. Wang, N. Ockman, M. Yan, R.R. Alfano [Solid-State Electron. (UK) vol.32 (1989) p.1337-45 ]
- [124] J.Y. Bigot, M.T. Portella, R.W. Schoenlein, J.E. Cunningham, C.V. Shank [Phys. Rev. Lett. (USA) vol.65 (1990) p.3429-32 ]
- [125] T. Elsaesser, J. Shah, L. Rota, P. Lugli [Semicond. Sci. Technol. (UK) vol.7 (1992) p.B144-7]
- [126] C.J. Stanton, D.W. Bailey [in 'Monte Carlo Device Simulation: Full Band and Beyond' Ed. K. Hess (Kluwer, Boston, 1991) p.67-97]
- [127] C.L. Collins, P.Y. Yu [ Phys. Rev. B (USA) vol.30 (1984) p.4501-15 ]
- [128] P.Lugli, L.Reggiani, M. Rieger, P. Kocevar [ Proc. 19th Int. Conf. on the Physics of Semiconductors, Warsaw, Poland, 15-18 Aug 1988, Ed. W. Zawadzki (Institute of Physics, Polish Academy of Sciences, Warsaw, Poland, 1988) vol.2 p.1465-72 ]
- [129] K. Brennan, K. Hess [Solid-State Electron. (UK) vol. 27 (1984) p.347-57]
- [130] K.F. Brennan, D.H. Park, K. Hess, M.A. Littlejohn [ J. Appl. Phys. (USA) vol.63 (1988) p.5004-8
- [131] S. Kratzer, J. Frey [ J. Appl. Phys. (USA) vol. 49 (1978) p.4064-8 ]
- [132] J.Pozela, A. Reklaitis [Solid-State Electron. (UK) vol.23 (1980) p.927-33]
- [133] M.C. Cheng, E.E. Kunhardt [ Solid State Commun. (USA) vol.79 (1991) p.651-5 ]
- [134] D.Y. Xing, M. Liu, C.S. Ting [Phys. Rev. B (USA) vol.37 (1988) p.10283-94]
- [135] M. Liu, D.Y. Xing, C.S. Ting [ J. Phys. Condens. Matter (UK) vol.1 (1989) p.407-18 ]
- [136] P.A. Houston, A.G.R. Evans [Solid-State Electron. (UK) vol.20 (1977) p.197-204]
- [137] I. Hase, K. Kawai, S. Imanaga, K. Kaneko, N. Watanabe [ J. Appl. Phys. (USA) vol.62 (1987) p.2558-60 ]
- [138] K. Berthold, A.F.J. Levi, J. Walker, R.J. Malik [ Appl. Phys. Lett. (USA) vol.54 (1989) p.813-5 ]
- [139] A.P. Dmitriev, M.P. Mikhailova, I.N. Yassievich [ Sov. Phys. Semicond. (USA) vol.17 (1983) p.28-32 ]
- [140] J. Vaitkus, A. Matulionis, L. Subacius, K. Jarasiunas [ Proc. 19th Int. Conf. on the Physics of Semiconductors, Warsaw, Poland, 15-18 Aug 1988, Ed. W. Zawadzki (Institute of Physics, Polish Academy of Sciences, Warsaw, Poland, 1988) vol.2 p.1447-50 ]
- [141] G.F. Glinskii, A.A. Kopylov, A.N. Pikhtin [ Solid State Commun. (USA) vol.30 (1979) p.613-4 ]
- [142] M.V. Fischetti [IEEE Trans. Electron Devices (USA) vol.38 634-49]

<sup>[143]</sup> P.J. Lin-Chung, K.L. Ngai [ Phys. Rev. Lett. (USA) vol.29 (1972) p.1610-13 ]

<sup>[144]</sup> K.L Ngai, E.J. Johnson [ Phys. Rev. Lett. (USA) vol.29 (1972) p.1607-10 ]

<sup>[145]</sup> W. Paul [ J. Phys. Chem. Solids (USA) vol.8 (1958), p.196-204 ]